



# Examiners' Report Principal Examiner Feedback

January 2020

Pearson Edexcel International GCE  
in Pure Mathematics P2 (WMA12) Paper 01

## **General**

This paper provided a range of challenges across all questions, though the good accessibility throughout that gave all students opportunity to show their ability. Perhaps the greatest difficulty came in understanding the context in question 5, with many not identifying the correct ratio required.

It is noteworthy as well that the majority of candidates were not prepared for the standard proof of the sum of an arithmetic series, with few fully correct attempts at part 8(a).

The advances of calculator technology created some problems for the marking of this paper, with the integral in question 4 and the summation in 8(ii) both being able to be achieved from the latest modern calculators with little working being shown. There is an expectation for full working to be shown, as noted on the front of the paper, so students would be well advised to ensure they show sufficient working for all their answers, including if their answer has been achieved by calculator.

Going forward questions will need to be clearer on when calculator technology is and is not appropriate, but the underlying understanding of students ought to be that reliance on calculators to avoid demonstrating understanding of the topics on the specification is not acceptable in most cases.

## **Question 1**

A very standard starting point in part (a) was followed by an immediate challenge of understanding of the relationship of integration and its approximation in part (b). Very few scored more than 5 marks in this question.

Part (a) was attempted well by the majority of candidates with correct use of the trapezium rule and achieving a correct answer of awrt 45.6. The most common mistakes were due to putting an extra term in the brackets or an incorrect  $h$  value using the formula with an incorrect value for  $n$ , usually  $h = \frac{14-2}{5}$ , despite the gaps between consecutive  $x$ -ordinates being clear from the table provided. A small number of students used  $x$  values, and some multiplied every term inside the brackets by 2 or multiplied instead of added terms.

Part (b) had a varied response by candidates and was a good discriminator across grades, with (i) being a challenge at E grade, while (ii) test A grade students. There were a few students who applied the trapezium rule again to both parts of (b), failing to appreciate how their part (a) answer could be used to calculate the required estimates.

In part (i) most candidates who used  $2\log_2(2x)$  went on to identify that this was  $2/5$  of their answer from part (a) and scored both marks, sometime due to the follow through mark. A few however integrated the factor of  $\frac{2}{5}$  to get  $\frac{2}{5}x$  and so lost accuracy. More common among lower grade students was to see an attempt at squaring the answer to (a) to form the answer.

For part (ii) even higher-grade students did not see how to proceed much of the time. Although many students did attain the method mark for attempting to isolate  $\log_2(2x)$  term, it was scored frequently via writing  $\log_2(2x) - \log_2(x^2)$  or  $\log_2(2) - \log_2(2x)$ , losing the accuracy mark in both cases. In the form cases either the trapezium rule to a calculator were used to find the integral of  $\log_2(x^2)$ .

The students who correctly identified the integrand as  $2 - \log_2(2x)$  generally went on to score both marks, although even here there were sometimes errors made.

Many students who attempted this part attempted to use  $\log_2\left(\frac{2}{x}\right) = \log_2 2 - \log_2 x$ . These usually were unable to make further progress, but a there some successful attempts which did this and also used  $\log_2(2x) = \log_2 2 + \log_2(x)$  with the answer from (a) to find the integral of  $\log_2 x$  first.

## **Question 2**

This question was another with good access in part (a) but a challenge for all students in part (b), with only about 20% scoring full marks.

In part (a) most students proceeded to write out the full expansion, not realising that they can use the general term formula to write the term in  $x^4$  directly. Only relatively few were able to isolate just the required term. However, the expansion was correct in most students work, and a correct equation in  $a$  achieved. Omission of the power on the  $a$  was rare, usually a common mistake, but mostly avoided for this paper.

The most common error in part (a) was to omit the negative root and give only  $\sqrt{2}$  as the solution, though this was often left as  $\sqrt[4]{4}$ . Another common error was to give only a decimal answer, and so lose the final accuracy mark. Indeed, the modal mark on this question was 3, usually scored as the first three marks of (a).

Attempts at taken out a factor of  $3^6$  first we infrequent, and in some cases led to an error in the coefficient when extracted from the series.

For part (b) many students perhaps struggled with the concept of the term independent of  $x$ , and many left this part blank, or instead tried to find the term in  $x$ . Others calculated only

$\frac{1}{81} \times \text{constant term from } (3+ax)^6$ , perhaps because they had only expanded up to the  $x^4$  term in part (a), so subsequently forgot the higher order terms. Many attempted a full expansion before trying to isolate terms independent of  $x$ .

Those who did realise that the term  $a^6 x^6 \times \frac{1}{x^6} = a^6$  was also independent of  $x$  often went on to score full marks, with few failing to add their answers. However a common mistake even among those attempt this was to omit the power 6 on the 3, result in  $\frac{1}{81} \times 3$  for the first constant term.

A small number of student lost the final A mark due to using a decimal value of  $a$  and failing to recover, while some others did achieve initially correct constant terms but made slips when adding them together.

### **Question 3**

This question proved much more accessible to students, seeming much more familiar territory than the first two questions, with most students scoring highly meaning this question was not a good discriminator for the paper.

In part (a) most students correctly showed that  $f\left(-\frac{3}{2}\right) = 0$  but many failed to write a conclusion

therefore scoring 1 out of 2 marks. A few students instead divided  $(2x + 3)$  into the polynomial and showed remainder was 0 which scored no marks as the questions asked candidates to use the factor theorem. It is important to read the question carefully to ensure the correct approach is taken.

Most students in part (b) successfully found the quadratic factor  $3x^2 + 4x - 4$ , usually by division, and were then able to proceed to a correct product of linear factors. However, a minority of students relied on their calculator's equation solving facilities to find the roots of  $f(x) = 0$  in order to write the expression as a product of linear factors. The question asked candidates to use algebra and therefore score no marks.

There were quite a number of students who used a tabular method with root  $-\frac{3}{2}$  and in effect

divided  $f(x)$  by  $\left(x + \frac{2}{3}\right)$  but then wrote  $(2x + 3)(6x^2 + 8x - 8) = (2x + 3)(3x - 2)(x + 2)$ , or

similar. Some credit was awarded to these, but to score full marks a fully correct solution was required.

For part (c) most students understood the link with part (b) and were able to correctly find an angle from at least one of their roots. However, only a minority were able to find both of the required angles. There was an issue with a number of students either not spotting that the question wanted answers in radians or unfamiliar with how to get angles in the required range from their principal angles. Where answers were given in radians, some students failed to gain the final answer mark due to an early rounding error where answers of 2.17 were given instead of 2.16.

### **Question 4**

This proved another accessible question with over half of students scoring full marks. The main two approaches were both seen frequently, with the difference between rectangle area and area under

curve being marginally more popular. Attempts using  $\int x \, dy$  were seen, albeit only in a small number of cases and were usually incorrectly carried out.

Most students were able to identify the limit  $\sqrt{5}$ , but a few used decimals in the integration in order to find the exact area, or failed to use their value in an integral or to find the area of a rectangle.

Performing the integration, when seen, was usually carried out correctly, with errors in the integral usually arising from bracketing errors in the "line – curve" approach. Only very few made errors in the coefficient or power or differentiated instead. However it is noteworthy that there were a number of students who relied on a calculator to evaluate the definite integral for them, which is not advisable when an exact answer is required as permitted calculators are supposed to only be able to carry out numerical integration, which will commonly give a decimal answer only as is not

acceptable method. As it happens, some calculators give the exact answer to this problem, and students were allowed some credit for this. But full method must be shown for full credit.

When using the “line – curve” approach, it was not uncommon to see the difference the wrong way round, though students would generally realise the final answer must be positive at the end and recover the marks.

One common error, aside from bracketing mistakes, made in this question was that only the area

under the curve being found,  $\frac{31\sqrt{5}}{3}$  being given as the area of the region  $R$ . Another mistake seen on numerous occasions was to use the  $y$  values of 7 and 17 as their limits to an integration with respect to  $x$ , which was incorrect and usually meant only the first two marks were scored.

A method seen occasionally was to split the area into a triangle and a small area between the curve and the diagonal line; some of those who tried this were successful.

### **Question 5**

This question proved challenging at all grades, and very few managed to score more than 5 marks in total. Understanding the problem at the start was the stumbling block for many, with incorrect ratios attempted, and very few were able to extract a percentage from their ratio correctly.

Indeed, in part (a) most students incorrectly used  $30000 r^2 = 34000$  instead of  $30000 r^3 = 34000$ , being unable to identify that three years after the start was in fact the fourth term in the sequence. The mark scheme was generous, and these candidates were still able to score 2 out of the available 3 marks for this part.

There were also some confused attempts trying to convert between the ratio,  $r$ , and the percentage,  $p$ . Many did not attempt the value of  $p$  as a percentage at all, thinking that  $p$  was 1.04, or 1.06 in the case where  $r^2$  was used.

In part (b) there were many students who incorrectly used the formulae for  $S_n$  rather than  $u_n$ , possibly because many "progression" questions in the past have tested knowledge of both the *term* and the *sum* formulas. Only 1 mark was available to these candidates, if they correctly solved their index equation using logs. This was another failing of comprehension of the question.

Also, many students who used  $N - 1$  (rather than  $N$ ) in part (b). These candidates could only score 3 of the 5 available marks in (b) unless they showed a correct understanding of the situation in their final answer (if they had  $r = \text{awrt } 1.04$ ). However, most students who realised the term formula was needed, whether with  $N$  or  $N - 1$ , were able to score the first two marks.

Most of the students did score the B mark for using logs correctly to solve an index equation and of those that had a fully correct equation, many went on to score the remaining two marks, for evaluating their log expression and rounding correctly.

The correct answer  $N = 22$  was rarely seen.

## **Question 6**

Nearly every candidate attempted this question, and it was extremely rare to see no marks awarded, but in many case progress did not go far beyond part (a). The topic of circles continues to be one that confounds many students, with the modal score being just the three marks scored in part (a). Part (a) was generally well answered. Those candidates who completed the square were usually successful in finding the correct coordinates of the centre although there were some sign errors, and again when dealing with the constant terms, resulting with an incorrect radius. But the correct coordinates and radius were achieved by most.

In part (b) those candidates who made a sketch were more successful to get the correct answer directly than the others who didn't make a sketch. However, it was not uncommon to see

$k = 2 \pm 3\sqrt{3}$  written directly, with or without a sketch.

A more common method, though, in this part was the Alternative Method of substituting  $y = k$  in the circle equation and setting the discriminant to zero then solving the resulting quadratic to find  $k$ . Some instead substituted  $x = -3$  and  $y = k$  at the same time and calculated the two values of  $k$ .

Attempts at solving via calculus were seldom seen, and usually incorrect attempts at the derivative were made, though correct attempts even via this alternative route were seen.

Some students added / subtracted the radius from the  $x$ -coordinate of the centre, when with a careful look at a diagram it would have been clear that the  $y$  coordinate was needed.

Far too common an approach, however, to part (b) was to substitute  $x = 0$  into the equation and attempt to solve, and this was probably more common than either approach above, as less than half of students accessed marks in part (b) or (c).

Part (c) was the most challenging part of the question and many students did not make any attempt to answer it at all. Others simply reiterated their answers to part (b). The students who made a sketch usually understood what the question asked and proceeded to answer correctly, but these were few and far between. Some did calculate the distance  $\sqrt{23}$  and go no further.

Substituting  $y = p$  and applying the difference of 4 between roots was rarely successful, instead the discriminant was commonly set equal to zero again by students attempting such a approach.

However, the approach of identifying either  $x = -5$  or  $-1$  as the required coordinate and solving the resulting quadratic, though rarely seen, was often successful when used.

### **Question 7**

This question, like question 3, was a good source of marks for students on this paper. In contrast to other questions where only very few scored highly, on this question less than 5 marks was uncommon. As such it did not discriminate particularly well across the grade boundaries.

In part (a) students were very well drilled in the method required with nearly all candidates using the identities  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  and  $\sin^2 \theta + \cos^2 \theta = 1$  in the equation. A few made errors in one or the other of these, so scored only 1 mark, but most were able to successfully complete the proof. Of those that reached the given equation by using the two correct identities, only a small number lost the final mark through notational errors, such as  $\cos \theta^2$  or by writing  $\frac{\sin}{\cos} \theta$  or not writing  $\theta$

Part (b) was answered well overall. The vast majority realised that they were required to use the result from part (a) and most obtained the critical value of  $\frac{1}{3}$ . Most of these used factors, although some used the quadratic formula, usually successfully. It was rare to see incorrect solutions to the quadratic.

Up to this point most responses were fully correct and it is only in the last two marks that a substantial number of errors were made. The second method mark required candidates to not only find the inverse sine of their critical value, but also to divide by 2 to get a value of  $x$ . A significant number failed to divide by 2 and so lost the final 2 marks. A few used inverse tan and so also lost these 2 marks.

Of those that used the correct method, a surprising number lost the Accuracy mark by either only giving one answer 9.74 or premature rounding error, stating one answer as 80.27, rather than 80.26. There were a small number that had an incorrect second critical value when solving the quadratic equation, but these could still score 3 out of the 4 marks in (b)

## **Question 8**

This question was not answered well by most of the candidates and few candidates gained full marks. Indeed, the modal mark on the question was one, being the B mark for  $u_5 = 22$  in part (ii)(a). The next most common mark was 4, usually scored by those who had learned the bookwork proof from part (i) and were able to score the marks.

In part (i) those who had learned the proof for the sum of the first  $n$  terms of an arithmetic series often gained full marks. It was, however, surprisingly unusual to see a well set out, fully correct proof. On the whole many candidates had a vague knowledge of how the proof started and where they wanted to get to, but a surprising proportion of candidates struggled to write a correct expression of  $S_n$  at the start of their proof, often having the last term incorrect, or they did not display a sufficient number of terms to make subsequent steps in their proof to complete. Others failed to reverse the terms in the sum correctly, or at all. In a few cases students tried to pair terms within one summation, (first + last) + (second + penultimate) + ... but they did not give due consideration to what happens when there are an odd number of terms, and were unable to score the accuracy mark as a result.

Some candidates either started with, or used something that they were trying to prove, e.g. starting with  $S_n = \frac{1}{2}n(a + L)$ , or attempting to use a summation formula that is a generalisation of the result they needed to show. Attempts at proof by induction were also seen in a few cases, though little progress was made in most of these (correct such responses were seen, though).

Students who used  $S_n = a + (a + d) + (a + 2d) + \dots + (L - d) + L$  rather than

$S_n = a + (a + d) + (a + 2d) + \dots + (a + (n - 1)d)$  were reasonably successful, as they were able to gain full marks providing they stated that  $L = a + (n - 1)d$  somewhere in their proof.

Overall, students would be well advised to make sure they know the standard proofs for the arithmetic and geometric series to ensure they are able to give should it be required.

In part (ii)(a) nearly all candidates gained the B mark by some showing calculation and some writing 22 without any working. This was often the only mark gained in part (ii).

Part(ii) (b) proved to be the most challenging part of the question, with a variety of approaches possible. Students usually began by writing out the first the terms of the summation, to try and spot the pattern, though many assumed it was a single arithmetic series, with variations on  $\frac{59}{2}(2 + 292)$

or  $\frac{59}{2}(2 \times 2 + (59 - 1) \times 11)$ , being seen often, leading to 8673 as an incorrect final answer;

otherwise first term (again) = 2, and common difference = 11 also was used, leading to all sorts of results.

Most correct solutions involved evaluating the sum on  $5n$  and the second part separately, achieving  $8850 - 3 = 8847$ . However, each of the variations in the scheme were seen on numerous occasions, with varying degrees of success with different ways of splitting it into two separate series seen;



once they realised that this was necessary, they usually made progress. Using the odd and even subsequences was relatively common, while pairing terms was less so, but done well once it had been spotted. Some also found the sum simply by evaluating the terms and adding.

It is unfortunate that this is another question where some students were able to procure the answer from a calculator and were able to gain marks for it, while others made valiant honest attempts to answer the question without success. Working is expected and answers without working may not gain full credit, so students ought to ensure they show working in any such question.

### **Question 9**

This question did see a spread of marks, though in part (b) it was very often a case of either one mark or all marks being scored. Most students were able to access some marks, but not many were scored highly.

In part (a) most were able to score both marks for their sketch, though some did drop a mark either through an incorrect sketch or failing to give the asymptote. However, there were many who made little or no attempt at a sketch at all. Some benefitted from the scheme being generous with the positioning of their horizontal asymptote, allowing a "gap" between the asymptote and the  $x$ -axis. Mistakes included sketching a negative exponential function and sometimes an intercept  $(0,1)$ .

For part (b) the majority of students understood that the  $x$ -coordinate of  $P$  was a root of the equation  $6^{1-x} = 3 \times 4^x$  and attempted to solve it using logarithms. Students who took the log of both sides of their initial equation often made mistakes in their manipulation of the subsequent log expressions, with  $\log(3 \times 4^x)$  causing most problems, including writing  $3 \times 4^x$  as  $12^x$ . In such case the first method mark was all that could be accessed.

A few omitted brackets when simplifying  $\log 6^{1-x}$ , writing  $1 - x \log 6$ , but these candidates could still score the method marks if they correctly applied the sum rule to the other side of their equation. Most that had achieved linear equation in  $x$  from suitable log work earned the method mark for rearranging to make  $x$  the subject and most of these that had a correct equation, went on to score the final accuracy mark by showing an intermediate step of working to the given answer.

Those who manipulated the powers before taking logs, generally did better than those who took logs initially. Of those attempting this method, some did not combine  $4^x$  and  $6^x$ , but many of these reached the given expression correctly by taking the log of  $(4^x \times 6^x)$  and returning to the main scheme by reaching a linear equation in  $x$ .

Some students instead proceeded from the equation in  $x$  to  $2 = 24^x$ , took logs and proceeded to the given expression. However, the step  $4^x \times 6^x = (4 \times 6)^x$  proved a stumbling block to a few, who failed to make further relevant progress before quoting the given answer.

Many showed  $\log_{10}$  in their working, but a number used just log, which was not penalised at all

### **Question 10**

Most students were able to access some marks in this question, though about 10% of students made little or no attempt and scored no marks. Being the last question on the paper it is possible that time may have been an issue for some, but overall this did not appear a problem on the paper. More likely is that students had given up by this stage, despite that the first part was relatively routine on this final question.

In part (a) the idea that a stationary point is where  $\frac{dy}{dx} = 0$  was shown to be understood by most students, who proceeded to attempt the derivative. The differentiation was of a very good standard, with only very few students making errors at this stage. Substitution of  $x = \frac{1}{2}$  usually followed, and in most cases it was seen set equal to zero. However, some attempted instead to use  $k = -\frac{3}{2}$  instead, so gained only the first two marks. The most common problems were not explicitly equating their  $\frac{dy}{dx}$  to zero; and failing to reach an answer via  $ax = b$  or  $ax + b = 0$ . Both of these omissions resulted in losing the final mark. For a given answer, sufficient evidence must be given to gain full credit. A few had algebraic or sign errors and so lost the final mark.

Where incorrect responses were seen, this was commonly because the student substituted  $x = \frac{1}{2}$  in the curve equation and calculated the corresponding  $y$  coordinate instead of using the derivative. Part (b) proved a bit more problematic for students. For those who did not successfully navigate part (a), there was usually no credit worthy work in part (b) either. At this stage those who substituted into the original equation would sometimes find the derivative at this stage. Others would instead at this stage return to the original equation and find the coordinates of the stationary point instead of finding its nature.

However, there were many who did recognise the second derivative was needed and proceeded to find it. Again the differentiation was usually done well, although the sign was often incorrect in the second term. It was not always clear if this was due to an error in substituting  $k$  or in differentiating, but in either case would usually lose only the accuracy. More of an issue with this part was that, having found the second derivative, not all students proceeded to evaluate at  $x = \frac{1}{2}$ , instead either using  $k$ , or simply making an unsubstantiated claim about the sign of the second derivative.

Those who achieved a value of  $-12$  for the second derivative at  $x = \frac{1}{2}$  would usually go on to make the correct deduction, though some omitted the reason.

Attempts at the first derivative test were seen, though not common. The closeness of the two stationary points often led to the attempts being unsuccessful, though if points  $x = 0.4$  and  $x = 0.6$  were chosen it was possible to answer correctly via this method.

In part (c) the question stated “using algebra” although this was ignored or not understood by a substantial number of students who again turned too quickly to their calculator to try and solve the

problem for them. In this case the question had given warning so they often would score no more than the first method mark.

Most students realised they needed to consider  $\frac{dy}{dx} = 0$  again and returned to  $12x^2 - 9 - kx^{-2} = 0$  but

many were not able to proceed to a 3 term quartic equation in  $x$ . Those who used their calculators to give 4 values from this equation gained no marks as a result, while those who reached the correct quartic equation, but then used their calculators and wrote the solutions of the equation directly and lost 3 marks.

Those who did realise that “using algebra” meant they needed to show their working make some progress if they reached a quartic. Commonly they changed the variable e.g.  $y = x^2$  for their quadratic equation, correctly solved it (mostly by factorisation) and stated the  $x$ -coordinate of the second stationary point. Some were able to achieve the factorisation without a substitution and also answer successfully, but there were also many who used “ $x^2 = x$ ” and end up solving the quadratic but failing to then square root the results.

Not everyone took notice of the part of the question where it stated  $x > 0$ . Giving both negative and positive values for the  $x$ -coordinate lost the final A mark.